

Dirac families for Loop groups as Matrix factorizations

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Abstract

We identify the category of integrable lowest-weight representations of the loop group LG of a compact Lie group G with the linear category of twisted, conjugation-equivariant *curved Fredholm complexes* on the group G : namely, the twisted, equivariant *matrix factorizations* of a super-potential built from the loop rotation action on LG . This lifts the isomorphism of K -groups of [FHT1]–[FHT3] to an equivalence of categories. The construction uses families of Dirac operators.

1. Introduction and background

The group LG of smooth loops in a compact Lie group G has a remarkable class of linear representations whose structure parallels the theory for compact Lie groups [PS]. The defining stipulation is the existence of a circle action on the representation, with finite-dimensional eigenspaces and spectrum bounded below, intertwining with the loop rotation action on LG . We denote the rotation circle by \mathbb{T}_r ; its infinitesimal generator L_0 represents the *energy* in a conformal field theory.

Noteworthy is the *projective nature* of these representations, described (when G is semi-simple) by a *level* $h \in H_G^3(G; \mathbb{Z})$ in the equivariant cohomology for the adjoint action of G on itself. The representation category $\mathfrak{Rep}^h(LG)$ at a given level h is semi-simple, with finitely many irreducible isomorphism classes. Irreducibles are classified by their *lowest weight* (plus some supplementary data, when G is not simply connected [FHT3, Ch. IV]).

In a series of papers [FHT1]–[FHT3] the authors, jointly with Michael Hopkins, construct $K^0\mathfrak{Rep}^h(LG)$ in terms of a twisted, conjugation-equivariant topological K -theory group. To wit, when G is connected, as we shall assume throughout this paper,¹ we have

$$K^0\mathfrak{Rep}^h(LG) \cong K_G^{\tau+\dim G}(G), \quad (1.1)$$

with a twisting $\tau \in H_G^3(G; \mathbb{Z})$ related to h , as explained below.

1.2 Remark. One loop group novelty is a *braided tensor* structure² on $\mathfrak{Rep}^h(LG)$. The structure arises from the *fusion product* of representations, relevant to 2-dimensional conformal field theory. The K -group in (1.1) carries a Pontryagin product, and the multiplications match in (1.1).

The map from representations to topological K -classes is implemented by the following *Dirac family*. Calling \mathcal{A} the space of connections on the trivial G -bundle over S^1 , the quotient stack $[G:G]$ under conjugation is equivalent to $[\mathcal{A}:LG]$ under the gauge action, via the holonomy map $\mathcal{A} \rightarrow G$. Denote by \mathbf{S}^\pm the (lowest-weight) modules of spinors for the loop space $L\mathfrak{g}$ of the Lie algebra and

¹Twisted loop groups show up when G is disconnected [FHT3].

²When G is not simply connected, there is a constraint on h .

by $\psi(A) : \mathbf{S}^\pm \rightarrow \mathbf{S}^\mp$ the action of a Clifford generator A , for $d + Adt \in \mathcal{A}$. A representation \mathbf{H} of LG leads to a family of Fredholm operators over \mathcal{A} ,

$$\mathcal{D}_A : \mathbf{H} \otimes \mathbf{S}^+ \rightarrow \mathbf{H} \otimes \mathbf{S}^-, \quad \mathcal{D}_A := \mathcal{D}_0 + i\psi(A) \quad (1.3)$$

where \mathcal{D}_0 is built from a certain Dirac operator $[L]$ on the loop group.³ The family is projectively LG -equivariant; dividing out by the subgroup $\Omega G \subset LG$ of based loops leads to a projective, G -equivariant Fredholm complex on G , whose K -theory class $[(\mathcal{D}_\bullet, \mathbf{H} \otimes \mathbf{S}^\pm)] \in K_G^{\tau+*}(G)$ is the image of \mathbf{H} in the isomorphism (1.1). When $\dim G$ is odd, $\mathbf{S}^+ = \mathbf{S}^-$ and skew-adjointness of \mathcal{D}_A leads to a class in K^1 . The twisting τ is the level of $\mathbf{H} \otimes \mathbf{S}$ as an LG -representation, with a (G -dependent) shift from the level h of \mathbf{H} .

The degree-shift is best explained in the world of super-categories, with $\mathbb{Z}/2$ gradings on morphisms and objects; odd simple objects have as endomorphisms the rank one Clifford algebra $\text{Cliff}(1)$, and contribute, in the semi-simple case, a free generator to K^1 instead of K^0 . Consider the τ -projective representations of LG with compatible action of $\text{Cliff}(L\mathfrak{g})$, thinking of them as modules for the (not so well-defined) crossed product $LG \ltimes \text{Cliff}(L\mathfrak{g})$. They form a semi-simple super-category \mathfrak{SRep}^τ , and the isomorphism (1.1) becomes

$$K^* \mathfrak{SRep}^\tau(LG \ltimes \text{Cliff}(L\mathfrak{g})) \cong K_G^{\tau+*}(G) \quad (1.4)$$

with the advantage of no shift in degree or twisting.⁴ This isomorphism is induced by the Dirac families of (1.3): a super-representation \mathbf{SH}^\pm of $LG \ltimes \text{Cliff}(L\mathfrak{g})$ can be coupled to the Dirac operators \mathcal{D}_\bullet without a choice of factorization $\mathbf{H} \otimes \mathbf{S}^\pm$.

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2. The main result

There is a curious lack of symmetry in (1.4): the isomorphism is induced by a functor of underlying Abelian categories, from $\mathbb{Z}/2$ -graded representations to twisted Fredholm bundles over G , but this functor is far from an equivalence. The category \mathfrak{SRep}^τ is semi-simple (in the graded sense discussed), but that of twisted Fredholm bundles is not so. We can even produce continua of non-isomorphic objects in any given K -class by perturbing a Fredholm family by compact operators.

Here, we redress this problem via the inclusion of a *super-potential* W , of algebraic geometry and 2-dimensional physics B -model fame. As explained by Orlov⁵ [O], this deforms the category of complexes of vector bundles into that of *matrix factorizations*: the *2-periodic, curved complexes* with curvature W . Our super-potential will have Morse critical points, leading to a semi-simple super-category with one generator for each critical point. The generators are precisely the Dirac families of (1.3) on irreducible LG -representations. This artifice of introducing a super-potential is redeemed by its natural topological origin, in the *loop rotation* action on the stack $[G:G]$. (Rotation is more evident in the presentation by connections, $[\mathcal{A}:LG]$.) Namely, for twistings τ transgressed from BG , the action of \mathbb{T}_r refines to a $B\mathbb{Z}$ -action on the G -equivariant *gerbe* G^τ over G which defines the K -theory twisting. The logarithm of this lift is $2\pi i W$.

³It is also the square root G_0 , in the super-Virasoro algebra, of the infinitesimal circle generator L_0 .

⁴For simply connected G , both sides live in degree $\dim \mathfrak{g}$; both parities can be present for general G .

⁵Orlov discusses complex algebraic vector bundles; we found no suitable exposition covering equivariant Fredholm complexes in topology, and a discussion is planned for our follow-up paper.

2.1 Remark. (i) The conceptual description of a super-potential as logarithm of a $B\mathbb{Z}$ -action on a category of sheaves is worked out in [P]; the matrix factorization category is the *Tate fixed-point category* for the $B\mathbb{Z}$ -action. On varieties, W is a function and $\exp(2\pi i W)$ defines the $B\mathbb{Z}$ -action; on a stack, there can be (as here) a geometric underlying action as well.

(ii) To reconcile our story with [P], we must rescale W_τ so that it takes integer values at all critical points; we will ignore this detail to better connect with the formulas in [FHT2, FHT3].

To spell out our construction, recall that a stack is an instance of a category. A $B\mathbb{Z}$ -action thereon is described by its generator, an automorphism of the identity functor. This is a section over the space of objects, valued in automorphisms, which is central for the groupoid multiplication. For $[G:G]$, the relevant section is the identity map $G \rightarrow G$ from objects to morphisms. Intrinsically, $[G:G]$ is the mapping stack from $B\mathbb{Z}$ to BG , and the $B\mathbb{Z}$ -action in question is the self-translation action of $B\mathbb{Z}$. This rigidifies the \mathbb{T}_τ -action on the homotopy equivalent spaces $LBG \sim BLG \sim \mathcal{A}/LG$.

A class $\hat{\tau} \in H^4(BG; \mathbb{Z})$ transgresses to a $\tau \in H_G^3(G; \mathbb{Z})$, with a natural \mathbb{T}_τ -equivariant refinement. This can also be rigidified, as follows. The exponential sequence lifts $\hat{\tau}$ uniquely to $H^3(BG; \mathbb{T})$, the group cohomology with smooth circle coefficients. That defines a Lie 2-group $G^{\hat{\tau}}$, a multiplicative \mathbb{T} -gerbe over G . (Multiplicativity encodes the original $\hat{\tau}$.) The mapping stack from $B\mathbb{Z}$ to $BG^{\hat{\tau}}$ is the quotient $[G^{\hat{\tau}}:G^{\hat{\tau}}]$ under conjugation, and carries a natural $B\mathbb{Z}$ -action from the self-translations of the latter. Because $B\mathbb{T} \hookrightarrow G^{\hat{\tau}}$ is strictly central, the self-conjugation action of $G^{\hat{\tau}}$ factors through G , and the quotient stack $[G^{\hat{\tau}}:G]$ is our $B\mathbb{Z}$ -equivariant gerbe over $[G:G]$ with band \mathbb{T} . We denote this central circle by \mathbb{T}_c , to distinguish it from \mathbb{T}_τ .

The $B\mathbb{Z}$ -action gives an automorphism $\exp(2\pi i W_\tau)$ of the identity of $[G^{\hat{\tau}}:G]$, lifting the one on $[G:G]$. Concretely, $[G^{\hat{\tau}}:G]$ defines a \mathbb{T}_c -central extension of the stabilizer of $[G:G]$, and $\exp(2\pi i W_\tau)$ is a trivialization of its fiber over the automorphism g at the point $g \in G$; see §3 below. The logarithm W_τ is multi-valued and only locally well-defined; nevertheless, the category of twisted matrix factorizations, $\mathrm{MF}_G^\tau(G; W_\tau)$ is well-defined, and its objects are represented by τ -twisted G -equivariant Fredholm complexes over G curved by $W_\tau + \mathbb{Z} \cdot \mathrm{Id}$.

2.2 Theorem. *The following defines an equivalence of categories from $\mathfrak{S}\mathfrak{R}\mathfrak{ep}^\tau$ to $\mathrm{MF}_G^\tau(G; W_\tau)$: a graded representation \mathbf{SH}^\pm goes to the twisted and curved Fredholm family $(\mathbb{D}_\bullet, \mathbf{SH}^\pm)$ whose value at the connection $d + A dt \in \mathcal{A}$ is the τ -projective LG -equivariant curved Fredholm complex*

$$\mathbb{D}_A = \mathbb{D}_0 + i\psi(A) : \mathbf{SH}^+ \rightleftarrows \mathbf{SH}^-$$

2.3 Remark. (i) Matrix factorizations obtained from irreducible representations are supported on single conjugacy classes, the so-called *Verlinde conjugacy classes* in G , for the twisting τ . These are the supports of the co-kernels of the Dirac families (1.3), [FHT3, §12].

(ii) There is a braided tensor structure on $\mathfrak{S}\mathfrak{R}\mathfrak{ep}^\tau(LG \ltimes \mathrm{Cliff}(\mathfrak{L}\mathfrak{g}))$ (without \mathbb{T}_τ -action). A corresponding structure on $\mathrm{MF}_G^\tau(G; W_\tau)$ should come from the Pontryagin product. We do not know how to spell out this structure, partly because the \mathbb{T}_τ -action is already built into the construction of MF^τ , and the Pontryagin product is *not* equivariant thereunder.

(iii) The values of the automorphism $\exp(2\pi i W_\tau)$ at the Verlinde conjugacy classes determine the *ribbon element* in $\mathfrak{R}\mathfrak{ep}^h(LG)$; see [FHLT] for the discussion when G is a torus.

Theorem 2.2 has a $\hat{\tau} \rightarrow \infty$ scaling limit, which we will use in the proof. In this limit, the representation category of LG becomes that of G . On the topological side, noting that each $\hat{\tau}$ defines an inner product on \mathfrak{g} , we zoom into a neighborhood of $1 \in G$ so that the inner product

stays fixed. This leads to a G -equivariant matrix factorization category $\mathrm{MF}_G(\mathfrak{g}, W)$ on the Lie algebra. The τ -central extensions of stabilizers near 1 have natural splittings, and the W_τ converge to a super-potential $W \in G \ltimes \mathrm{Sym}(\mathfrak{g}^*)$, which, in a basis ξ_a of \mathfrak{g} with dual basis ξ^a of \mathfrak{g}^* , we will calculate in §3 to be

$$W = -i \cdot \xi_a(\delta_1) \otimes \xi^a + \frac{1}{2} \sum_a \|\xi^a\|^2 \quad (2.4)$$

with $\xi_a(\delta_1)$ denoting the respective derivative of the delta-function at $1 \in G$. It is important that (2.4) is central in the crossed product algebra $G \ltimes \mathrm{Sym}(\mathfrak{g}^*)$.

To describe the limiting case, recall from [FHT3, §4] the G -analogue of the Dirac family (1.3). Kostant's *cubic Dirac operator* [K] on G is left-invariant, and the Peter-Weyl decomposition gives an operator $\mathcal{D}_0 : \mathbf{V} \otimes \mathbf{S}^\pm \rightarrow \mathbf{V} \otimes \mathbf{S}^\mp$ for any irreducible representation \mathbf{V} of G , coupled to the spinors \mathbf{S}^\pm on \mathfrak{g} . As before, it is better to work with graded modules for the super-algebra $G \ltimes \mathrm{Cliff}(\mathfrak{g})$.

2.5 Theorem. *Sending \mathbf{SV}^\pm to $(\mathcal{D}_\bullet, \mathbf{SV}^\pm)$, the curved complex over \mathfrak{g} given by*

$$\mathfrak{g} \ni \mu \mapsto \mathcal{D}_\mu = \mathcal{D}_0 + i\psi(\mu) : \mathbf{SV}^+ \rightleftharpoons \mathbf{SV}^-$$

provides an equivalence of super-categories from graded $G \ltimes \mathrm{Cliff}(\mathfrak{g})$ -modules \mathbf{SV}^\pm to G -equivariant, W -matrix factorizations over \mathfrak{g} .

With λ denoting the lowest weight of V and $T(\mu)$ the μ -action on \mathbf{SV} , we have [FHT3, Cor. 4.8]

$$\mathcal{D}_\mu^2 = -\|\lambda_V + \rho\|^2 + 2i \cdot T(\mu) - \|\mu\|^2 \in (-2W) + \mathbb{Z}.$$

3. Outline of the proof

(3.1) *Executive summary.* The category $\mathrm{MF}_G^\tau(G; W_\tau)$ sheafifies over the conjugacy classes of G . Near any $g \in G$ with centralizer Z , the stack $[G : G]$ is modeled on a neighborhood of 0 in the adjoint quotient $[\mathfrak{z} : Z]$ of the Lie algebra \mathfrak{z} , via $\zeta \in \mathfrak{z} \mapsto g \cdot \exp(2\pi\zeta)$. We will compute the local W_τ in the crossed product $Z \ltimes C^\infty(\mathfrak{z})$, recovering (2.4), up to a g -dependent central translation in \mathfrak{z} . We then show that MF^τ lives only on *regular* elements g . Assuming for brevity that $\pi_1(G)$ is torsion-free: Z is then the maximal torus $T \subset G$, where the super-potential W_τ turns out to have Morse critical points, located precisely at the Verlinde conjugacy classes. The local category is freely generated by the respective Atiyah-Bott-Schapiro Thom complex.⁶ The latter is quasi-isomorphic to our Dirac family for a specific irreducible representation, associated with the Verlinde class [FHT3, §12].

(3.2) *The 2-group.* We use a *Whitehead crossed module* [W] description for $G^\hat{\tau}$. This is an exact sequence of groups

$$\mathbb{T}_c \hookrightarrow K \xrightarrow{\varphi} H \twoheadrightarrow G,$$

equipped with an action of H on K which lifts the self-conjugation of H and factors the self-conjugation of K via φ . Call h an H -lift of g , and C the pre-image of Z in H . Define the central extension \tilde{Z} by means of a \mathbb{T}_c -central extension of C , trivialized over $\varphi(K) \cap C$, as follows. The commutator $c \mapsto hch^{-1}c^{-1}$ gives crossed homomorphism $\chi : C \rightarrow \varphi(K)$, with respect to the conjugation action of C on $\varphi(K)$. The action having been lifted to K , χ pulls back the central extension $\mathbb{T}_c \hookrightarrow K \rightarrow \varphi(K)$ to C . The h -action on K identifies the fiber of K over any $c \in \varphi(K)$ with that over hch^{-1} , trivializing the pull-back extension over $\varphi(K)$. Finally, $hhh^{-1}h^{-1} = 1$, so the extension is also trivialized over $c = h$, defining our $\exp(2\pi i W_\tau)$ at $g \in Z$.

⁶The Clifford multiplication acts in both directions, giving a curved complex.

(3.3) *Computing the local super-potential.* Following [BSCS], take $K = \Omega^\tau G$, the τ -central extension of the group of based smooth maps $[0, 2\pi] \rightarrow G$ sending $\{0, 2\pi\}$ to 1, and $H = \mathcal{P}_1 G$, the group of smooth paths $[0, 2\pi] \rightarrow G$ starting at $1 \in G$. The requisite H -action on the Lie algebra $i\mathbb{R} \oplus \Omega\mathfrak{g}$ of K is

$$\gamma.(x \oplus \alpha) = \left(x - \frac{i}{2\pi} \int_0^{2\pi} \langle \gamma^{-1} d\gamma | \alpha \rangle \oplus \text{Ad}_\gamma(\alpha) \right) \quad (3.4)$$

extending the Ad-action of $\Omega^\tau G$ [PS, Prop. 4.3.2], and exponentiating to an action on $\Omega^\tau G$. (Acting on other components of ΩG requires topological information from $\hat{\tau}$.)

The equivariant gerbe $[G^{\hat{\tau}} : G]$ is locally trivialized (possibly on a finite cover of Z) uniquely up to discrete choices: the automorphisms of the central extension \tilde{Z} . We spell out W_τ in these terms. Lift g to $\mathcal{P}_1 G$ as $h = \exp(t\mu)$, for a shortest logarithm $2\pi\mu$ of g , and assume for now that Z centralizes μ . Instead of the entire group C , we use in the construction the subgroup $\mathcal{P}_1 Z$ of paths in Z . It centralizes h , and this trivializes our \mathbb{T}_c -extension over $\mathcal{P}_1 Z$, with $W_\tau = 0$. However, by (3.4), the extension over $\Omega Z = \varphi(K) \cap \mathcal{P}_1 Z$ is trivialized by the Lie algebra character $\alpha \mapsto -\frac{i}{2\pi} \int_0^{2\pi} \langle \mu | \alpha \rangle dt$. To trivialize \tilde{Z} , we must therefore extend this to a linear character of $\mathcal{P}_0 \mathfrak{z}$. The same formula (3.4) does this, supplying the locally constant trivialization of \tilde{Z} . We now get the value $2\pi i W_\tau(g) = \pi i \|\mu\|^2 \oplus 2\pi\mu \in i\mathbb{R} \oplus \mathfrak{g}$.

At the remaining points, W_τ is determined by continuity, but can also be pinned down by the restriction to a maximal torus containing g .

(3.5) *Vanishing of singular contributions.* When \mathfrak{z} is non-abelian, we show the vanishing of the matrix factorization category localized at g . Take $g = 1, Z = G, W$ on \mathfrak{g} as in (2.4), plus possibly a central linear term μ . Koszul duality equates the localized category $\text{MF}_G^\tau(\mathfrak{g}; W)$ with the super-category of $\mathbb{Z}/2$ -graded modules over the differential super-algebra $(G \ltimes \text{Cliff}(\mathfrak{g}), [\mathcal{D}_\mu, _])$; $\mathcal{D}_\mu = \mathcal{D}_0 + i\psi(\mu)$, with Kostant's cubic Dirac operator of §2. Ignoring the differential, the algebra is semi-simple, with simple modules the $\mathbf{V} \otimes \mathbf{S}^\pm$ of Theorem 2.5, for the irreducible G -representations \mathbf{V} . Since $\mathcal{D}_\mu^2 = -\|\lambda_V + \mu + \rho\|^2 < 0$, $[\mathcal{D}_\mu, \mathcal{D}_\mu]$ provides a homotopy between 0 and a central unit in the algebra. This makes the super-category of graded modules quasi-equivalent to 0.

(3.6) *Globalization for the torus.* We describe the stack $[T^{\hat{\tau}} : T]$ and potential W_τ in the presentation $T = [\mathfrak{t} : \Pi]$ of the torus as a quotient of its Lie algebra by $\Pi \cong \pi_1(T)$. Lifted to \mathfrak{t} , the gerbe of stabilizers \tilde{T} is trivial, with band $T \times \mathbb{T}_c$. The descent datum under translation by $p \in \Pi$ is the shearing automorphism of $T \times \mathbb{T}_c$ given by the character $t \mapsto \exp\langle p | \log t \rangle$, $t \in T$. In the same trivialization over \mathfrak{t} , the super-potential is

$$2\pi i W_\tau(\mu) = \pi i \|\mu\|^2 \oplus 2\pi\mu \in i\mathbb{R} \oplus \mathfrak{t},$$

the first factor being the Lie algebra of \mathbb{T}_c . That is the function “ $\frac{1}{2}\|\log\|^2$ ” on T , invariant under p -translation, save for an additive shift by the integer $\|p\|^2/2$.

With Λ denoting the character lattice of T , the crossed product algebra of the stack $[T^\tau : T]$ can be identified with the functions on $(\coprod_{\lambda \in \Lambda} \mathfrak{t}_\lambda) / \Pi$, with the action of Π by simultaneous translation on Λ and \mathfrak{t} . On the sheet $\lambda \in \Lambda$, $W_\tau = -\langle \lambda | \mu \rangle + \|\mu\|^2/2$ has a single Morse critical point at $\mu = \lambda$.

It follows that the super-category $\text{MF}_T^\tau(T; W_\tau)$ is semi-simple, with one generator of parity $\dim \mathfrak{t}$ at each point in the kernel of the isogeny $T \rightarrow T^*$ derived from the quadratic form $\hat{\tau} \in H^4(BT; \mathbb{Z})$. The kernel comprises precisely the Verlinde points in T [FHLT], and this concludes the proof.

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